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Name of College - S.S. College Jehanabad

Topic - Sequence and their Convergence

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~~B.Sc~~ BCA I Date - 12 - 04 - 2020

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* Sequence :→ A function u whose domain is the set of Natural numbers N is called a sequence. Thus a real sequence u is a correspondence which associates each natural number to uniquely determined real number. The image of the natural number n is denoted by u_n or $u(n)$.

usual Notation of sequence is denoted by $\{u_n\}$

u_n = nth Term of the sequence.

Ex. → (i) $1, 3, 5, 7 \ldots, 2n-1, \ldots$ is a sequence for which $u_1 = 1$ $u_2 = 3$ .. $u_n = 2n-1$ etc.

(ii) $-1, +1, -1, +1 \ldots$ is a sequence for which

$u_1 = (-1)$ $u_3 = -1 = (-1)^3$

$u_2 = +1 = (-1)^2$ $u_4 = 1 = (-1)^4$

$u_n = (-1)^n$ — — —

The set $\{x \mid x = u_n, n \in N\}$ of distinct terms of the sequence $\{u_n\}$ is called the range set of the sequence $\{u_n\}$

$u = \{u_1, u_2, u_3 \ldots, u_n, \ldots\}$

Ex. → The Range set of $\{u_n\}$, $u_n = 3n+1$

$u_1 = 3\times1+1 = 4$ $u_2 = 3\times2+1 = 7$ $u_3 = 3\times3+1 = 10$

The range set $= \{4, 7, 10, \ldots\}$ which is a subset of R.

Bounded Sequence :→ The Sequence $\{u_n\}$ is said to be bounded above if there exists a finite Number $K \in R$ such that

$u_n \leq K$ for all values of $n \in N$.

The Sequence $\{u_n\}$ is said to be bounded below if there exists a finite no $k \in R$ such that $u_n \geq k$ for all values of $n \in N$.

If the Sequence is bounded above and below is said to be bounded, and the K and k are called rough upper and lower bounds respectively.

$\therefore$ u_n is bounded sequence if

$k \leq u_n \leq K \quad \forall n \in N$

Least upper bounds and Greatest lower bounds:→

If M is a Number such that

(i) $u_n \leq M \quad \forall n \in N$

and (ii) $u_n > M - \epsilon$, for at least one value of $n \in N$

Where ϵ is an arbitrary +ve number, however small, then M is called the least upper bound of the sequence $\{u_n\}$, which is bounded above.

If m is a number such that

(i) $u_n \geq m$ for all values of n

and (ii) $u_n < m + \epsilon$, for at least one value of $n \in N$

Where ϵ is an arbitrary +ve number, however small, then m is called the Greatest lower bound

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Monotonic Sequence :→

If $U_{n+1} \geq U_n$ for all values of $n \in N$ Sequence $\{U_n\}$ is said to be a monotonic increasing sequence.

for a monotonic sequence $\{u_n\}$

$u_1 \leq u_2 \leq u_3 \leq \dots \leq u_n \leq \dots$

If $U_{n+1} \leq U_n$ for all values of $n \in N$ then U_n is said to decrease steadily and the sequence $\{U_n\}$ is said to be a monotonic decreasing sequence.

Limit of a Sequence →

The sequence $\{U_n\}$ is said to have the limit l if for a given positive number ϵ there exists a positive integer m such that $|U_n - l| < \epsilon$ for all integral values of $n \geq m$.

If $\{u_n\}$ has the limit l we show it by writing

$$\lim_{n \to \infty} U_n = l$$

Convergence of Sequence :→

If $\lim_{n \to \infty} U_n = l$ then we say sequence $\{U_n\}$ is convergent.

Ex :→ let us consider a sequence whose nth term $U_n = \frac{2n-1}{2n}$ is convergent

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Here $u_n = \frac{2n-1}{2n}$

Now $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{2n-1}{2n}$

$= \lim_{n\to\infty} \frac{n(2-1/n)}{2n} = \frac{2}{2}$

As $n \to \infty \Rightarrow 1/n \to 0$

Thus $\{u_n\}$ is convergent where $u_n = \frac{2n-1}{2n}$.

Divergent Sequence:→

If $\lim_{n\to\infty} u_n = \infty$ or

$\lim_{n\to\infty} u_n = -\infty$

then Sequence $\{u_n\}$ is Called Divergent.